9/15/16 "Quality is not an act, it is a habit." -Aristotle

HW: "Infinite Limits and Limits at Infinity HW"

Test 1 on Tuesday 9/20

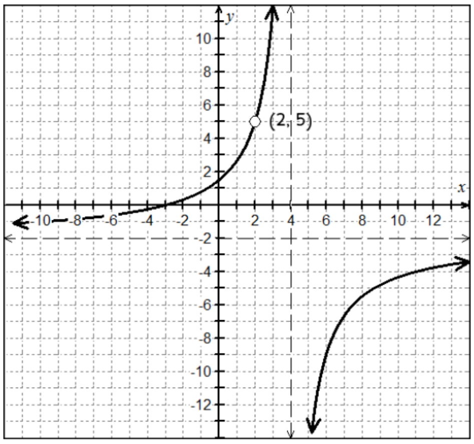
AIM: Limits at Infinity

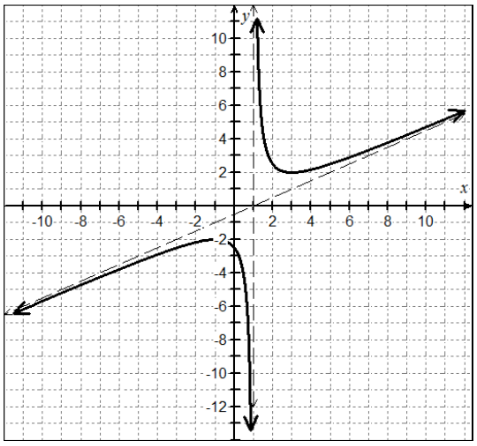
Infinite Limits and Limits at Infinity

In the lesson on *Understanding Limits* you were confronted with these two situations. Here we begin to **compare and contrast** the behavior of functions as they *approach infinity*, as well as, functions that *tend toward infinity* in certain circumstances. Let's go…

EX #1: Use the graphs of $f(x)$ and $g(x)$ shown below to compare and contrast behaviors involving infinity. Let's go...

$$f(x)=\frac{-2x^2-2x+12}{x^2-6x+8} \qquad g(x)=\frac{x^2-2x+5}{2x-2}$$





An Infinite Limit is:	A Limit at Infinity is:
a limit $x \to a$ results in ∞ or $-\infty$	a limit where $x \to \infty$ or $x \to -\infty$
$\lim_{x\to 4^-} f(x) =$ ∞	$\lim_{x\to -\infty} f(x) =$ -2
$\lim_{x\to 4^+} f(x) =$ $-\infty$	$\lim_{x\to \infty} f(x) =$ -2
$\lim_{x\to 1^-} g(x) =$ $-\infty$	$\lim_{x\to -\infty} g(x) =$ $-\infty$
$\lim_{x\to 1^+} g(x) =$ ∞	$\lim_{x\to \infty} g(x) =$ ∞
Discovery: Vertical Asymptote	Discovery: Horizontal or Oblique Asymptotes (End Behavior Asymptotes)

In Pre-Calculus you learned some basic truths about rational functions.

1. When a factor cancelled from the denominator a ____hole____ occurred.
2. When a factor would not cancel from the denominator a ____vertical asymptote____ occurred.

EX #2: Use the previous equations to find the limits analytically.

A. $\lim_{x \to 4^-} \frac{-2x^2 - 2x + 12}{x^2 - 6x + 8}$

Definition and Justification of Vertical Asymptotes

Case 1: $h(c)=\frac{non-zero}{zero}$ und.	Case 2: $h(c)=\frac{zero}{zero}$ inderminant
$x=c$ is: Vertical Asymptote	$x=c$ is: hole

IN CALCULUS, YOU MUST USE NEW LANGUAGE IN ORDER TO JUSTIFY!

LIMIT DEFINITION (JUSTIFICATION) OF A VERTICAL ASYMPTOTE

If $\lim\limits_{x \to a^-} f(x) = \pm\infty$	If $\lim\limits_{x \to a^+} f(x) = \pm\infty$
$x=a$ is vertical asymptote	also $x=a$ is a V.A.

Limits at Infinity

Next, we will explore **limits at infinity** in order to differentiate between the two conditions. Recall the lessons from Pre-Calculus related to analyzing the **end behavior of functions.** In the exercise below, use this prior knowledge to find each limit at infinity.

EX #4: Find each limit at infinity, explain your thinking.

x^4

A. $\lim_{x \to \infty} (x^2 - 4)(x^2 + 3)$ ← Degree 4

⊛ If the degree is even then look at the lead coefficient

if its positive as $x \to \infty$ Limit is ∞

$x \to -\infty$ Limit is ∞

If its negative then the reverse is true

B. $\lim_{x \to -\infty} (5x^3 - 2x + 4)$

If degree is odd

$x \to -\infty$ limit is $-\infty$

$x \to \infty$ limit is ∞

unless the lead coefficient is negative then it reverses.

C. $\lim_{x\to\infty} \frac{3x^2-4}{x^2+1}$

Degree of Numerator = Degree of denominator

limit = $\frac{3}{1}$

Then there is a horizontal asymptote.

The limit = $\frac{\text{lead coefficient of num.}}{\text{lead coefficient of den.}}$

D. $\lim_{x\to-\infty} \frac{5x-2}{x^2+1}$

Degree of Numerator < Degree of Denominator

limit = 0

There is a horizontal asymptote at $y=0$

Revisiting the rules for finding potential horizontal asymptotes for rational functions from Pre-Calculus, you can use the idea of a limit and calculus to see why those rules hold true.

1. If degree of numerator is less than degree of denominator (bottom heavy), then limit is zero.

$$\lim_{x\to\pm\infty} f(x) = 0$$

2. If degree of numerator equals degree of denominator (powers equal), then limit is the ratio of coefficients of the highest degree.

$$\lim_{x\to\pm\infty} f(x) = \frac{\text{coefficient of numerator's highest power}}{\text{coefficient of denominator's highest power}}$$

3. If degree of numerator is greater than degree of denominator (top heavy), then limit does not exist.

$$\lim_{x\to\pm\infty} f(x) = \pm\infty$$